Ans Importance of Lagrangian equation :->

The Newtonian and Lagrangian equation of motion are second order differential equation of motion of the system. When these equation are solved. They discribe the nature of motion have different approch to formulation.

In Newtonian approch, we are Consermed with the applied forces acting on the system. This forces produces acceleration in the stem. The force are due to enternal energy. These forces represented the cause of acceleration.

In Lagrangian apporch, we consider the kinetic energy and potential energy of the system. The concept of forces does not inter the Lagrangian formulation. This is the difference between Newtonian and Lagrangian approch. Since kinetic energy and potential energy are scelar quantity. Therefore they are invariment under coordinate transformation. Another diffrence is that insome problem. It is not possible to know all forces acting on the system. These forces are of constraints. These constraints are not removed in Newton formula. But Lagrangian equation of motion is expressed in the term of generalised

Coordinate. Hence forces are constraints are easly removed.

$$\frac{dq_J}{dt} = \dot{q}_J = \text{Generalised velocity.}$$

$$\dot{x} = \frac{dx}{dt} = v$$

$$\therefore T = \frac{1}{2}mv^2 = \frac{1}{2}m\dot{x}^2$$

$$\frac{\partial T}{\partial \dot{x}} = m\dot{x} = P_x = \text{Momentum}$$

$$\therefore \frac{\partial T}{\partial \dot{q}_J} = P_J = \text{Generalised Momentum}$$

$$\frac{d}{dt} \cdot \frac{\partial T}{\partial \dot{q}_J} - \frac{\partial T}{\partial q_J} = Q_J$$

$$\frac{d}{dt} \frac{\partial T}{\partial \dot{q}_J} - \frac{\partial L}{\partial q_J} = 0$$

$$\frac{\partial L}{\partial \dot{q}_J} = \frac{\partial T}{\partial \dot{q}_J} = P_J = \text{Generalised momentum.}$$

$$\frac{dP_J}{dt} - \frac{\partial L}{\partial q_J} = 0$$

$$\dot{P}_J - \frac{\partial L}{\partial q_J} = 0$$

$$\frac{\partial L}{\partial q_J} = \dot{P}_J$$

$\dot{q}_J$ = Generalised velocity

$$\frac{\partial L}{\partial \dot{q}_J} = \frac{\partial T}{\partial \dot{q}_J} = P_J \text{ Generalised Momentum}$$

$$\frac{\partial L}{\partial q_J} = \frac{d}{dt} \frac{\partial L}{\partial \dot{q}_J} = \dot{P}_J = \text{Rate of change of Generalised momentum.}$$

If Lagrangian of a system is explicitly Independent of time.

i-e. For a conservative system

$$L = L(q_J, \dot{q}_J, t)$$

$$\therefore \frac{dL}{dt} = \sum_J \frac{\partial L}{\partial q_J} \cdot \frac{dq_J}{dt} + \sum_J \frac{\partial L}{\partial \dot{q}_J} \frac{d\dot{q}_J}{dt}$$

$$\frac{dL}{dt} = \sum_J \frac{\partial L}{\partial q_J} \cdot \dot{q}_J + \frac{\partial L}{\partial \dot{q}_J} \cdot \ddot{q}_J$$

But from Lagrangian equation

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_J} - \frac{\partial L}{\partial q_J} = 0$$

$$\therefore \frac{\partial L}{\partial q_J} = \frac{d}{dt}\frac{\partial L}{\partial \dot{q}_J}$$

Combaning equation (1) and (11) we have

$$\frac{dL}{dt} = \sum_J \frac{d}{dt}\frac{\partial L}{\partial \dot{q}_J}\cdot \dot{q}_J + \sum_J \frac{\partial L}{\partial \dot{q}_J}\ddot{q}_J$$

$$\text{or, } \frac{dL}{dt} = \sum_J \frac{d}{dt}\left\{\frac{\partial L}{\partial \dot{q}_J}\cdot \dot{q}_J\right\}$$

$$\text{or, } \frac{dL}{dt} = \frac{d}{dt}\sum_J \frac{\partial L}{\partial \dot{q}_J}\dot{q}_J$$

$$\text{or, } \frac{d}{dt}\sum_J \frac{\partial L}{\partial \dot{q}_J} - \frac{dL}{dt} = 0$$

$$\text{or, } \frac{d}{dt}\left[\sum_J \frac{\partial L}{\partial \dot{q}_J}\cdot \dot{q}_J - L\right] = 0$$

$$\therefore \sum_J \frac{\partial L}{\partial \dot{q}_J} - L = H \quad \text{(III)}$$ Where H is a constant

$$\text{or, } \sum_J \frac{\partial L}{\partial \dot{q}_J}\cdot \dot{q}_J - L = H$$ where H is called Hamiltonian of the system

$$\text{or, } H = \sum_J P_J \cdot \dot{q}_J - L(q_J, \dot{q}_J, t)$$

$$\therefore H = H(q_J, P_J, t)$$

Thus Hamiltonian of a system of function of generalised coordinate, generalised momenta and time.

$$\therefore \sum \frac{\partial L}{\partial \dot{q}_J}\cdot \dot{q}_J - L = H$$

$$\text{or, } \frac{\partial T}{\partial \dot{q}_J}\cdot \dot{q}_J - L = H$$

$$\text{or, } \dot{q}_J \cdot \frac{\partial T}{\partial \dot{q}_J} - L = H$$

From Euler's theorem.

$$\dot{q}_J \frac{\partial T}{\partial \dot{q}_J} = 2T$$

$$\therefore 2T - L = H$$

$$\text{or, } 2T - (T - V) = H \qquad \{\because L = T - V\}$$

$$\text{or, } 2T - T + V = H$$

$$\therefore T + V = H$$

K.E + P.E = H (constant)

Thus Hamiltonian of a system is total